Benchmarking calculations of the magnetic field near the last closed flux surface with finite plasma pressure and current

- J.C. Schmitt[1], A. Bader[2], M.R. Cianciosa[3], M. Drevlak[4], H. Frerichs[2], S.A. Lazerson[5], J.D. Lore[3], and D. Maurer[1]
- *1. Auburn University, Department of Physics, Auburn, AL, 36849, USA*
- *2. University of Wisconsin, Engineering Physics, Madison, WI, 53706, USA*
- *3. Oak Ridge National Laboratory, Oak Ridge, TN, 37831, USA*
- *4. Max-Planck-Institut für Plasmaphysik, D17491-Greifswald, Germany*
- *5. Princeton Plasma Physics Laboratory, Princeton, NJ, 08540, USA*

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Abstract

The calculation of the magnetic field near the last closed flux surface in the presence of finite plasma pressure and current is important because of its impact on subsequent calculations. For instance, subtle differences in the calculation of the magnetic field leads to differences in magnetic field line trajectories. These field line trajectories are subsequently used to help generate a 'field-aligned' grid for edge transport and divertor modelling with the EMC3-EIRENE code [1,2]. A goal for this work is to understand the impact the magnetic field calculations have on the resulting field-aligned grid.

Two methods of magnetic field calculations with finite plasma pressure and current are discussed. One method is based on the virtual casing (VC) theorem and the other uses the magnetic vector potential. The DIAGNO [3] and EXTENDER [4] codes implement the VC theorem, while the BMW [5] code uses the magnetic vector potential. For both methods, a plasma equilibrium provided by VMEC is provided. Differences in the magnetic field and subsequent calculations for W7-X configurations will be presented and discussed. This work will help determine the strategy on how best to pass reconstructed equilibrium information from V3FIT to EMC3/EIRENE.

This work is supported by U.S. Department of Energy grant DE-SC00014529.

References

[1] Y. Feng, F. Sardei, J. Kisslinger, and P. Grigull, J. Nucl. Mater., vols. 241–243, pp. 930-934 (1997).

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[3] S. A. Lazerson, S. Sakakibara, and Y. Suzuki, Plasma Phys. Control. Fusion **55**, 025014 (2013).

[4] M. Drevlak, D. Monticello, and A. Reiman, Nuclear Fusion, **45**, (2005).

[5] M.R. Cianciosa, et al., Bulletin of the 58th Annual Meeting of the APS Division of Plasma Physics, Vol 61, Number 18, GP10.00060, October 31–November 4 2016; San Jose, California.

Magnetic fields calculations with multiple sources

- The magnetic field vector **B** at a position **r**, is given by

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \oint_C I \frac{d\boldsymbol{l} \times \boldsymbol{r}'}{|\boldsymbol{r}'|^3}$$

- Sufficient for calculating the magnetic field in vacuum when the only source of current is from coils
  - Time-varying currents and the presence of materials with relative permeability $\neq 1$ alter the fields
- The field vector can also be calculated with the curl of the magnetic field potential

$$\boldsymbol{B}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r}) \qquad \boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \iiint_V d^3\boldsymbol{r}' \frac{\boldsymbol{J}(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|}$$

Calculations of magnetic field in presence of plasma currents, I

- Virtual casing theorem
  - Instead of calculating the field due to a volume of current densities, an equivalent surface current density and dipole moment density (on the surface of the plasma) is found

$$\boldsymbol{K} = -\frac{1}{\mu_0}\hat{n}\times\boldsymbol{B} \qquad \sigma_{dipole} = -\hat{n}\cdot\boldsymbol{B}$$

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\int dA'\,\frac{\boldsymbol{K}\times(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}\prime|^3} + \frac{1}{4\pi}\int dA'\,\frac{\sigma_{dipole}(\boldsymbol{r}-\boldsymbol{r}')}{|\boldsymbol{r}-\boldsymbol{r}\prime|^3}$$

  - Due to finite accuracy of numerical integration near the boundary, adaptive algorithm are necessary
  - Implemented in EXTENDER and FIELDLINES for arbitrary grids

M. Drevlak, D. Monticellow and A. Reiman, Nuclear Fusion **45** (2005) 731
S.A. Lazerson, Plamsa Phys. Control. Fusion **54** (2012) 122002

Calculations of magnetic field in presence of plasma currents, II

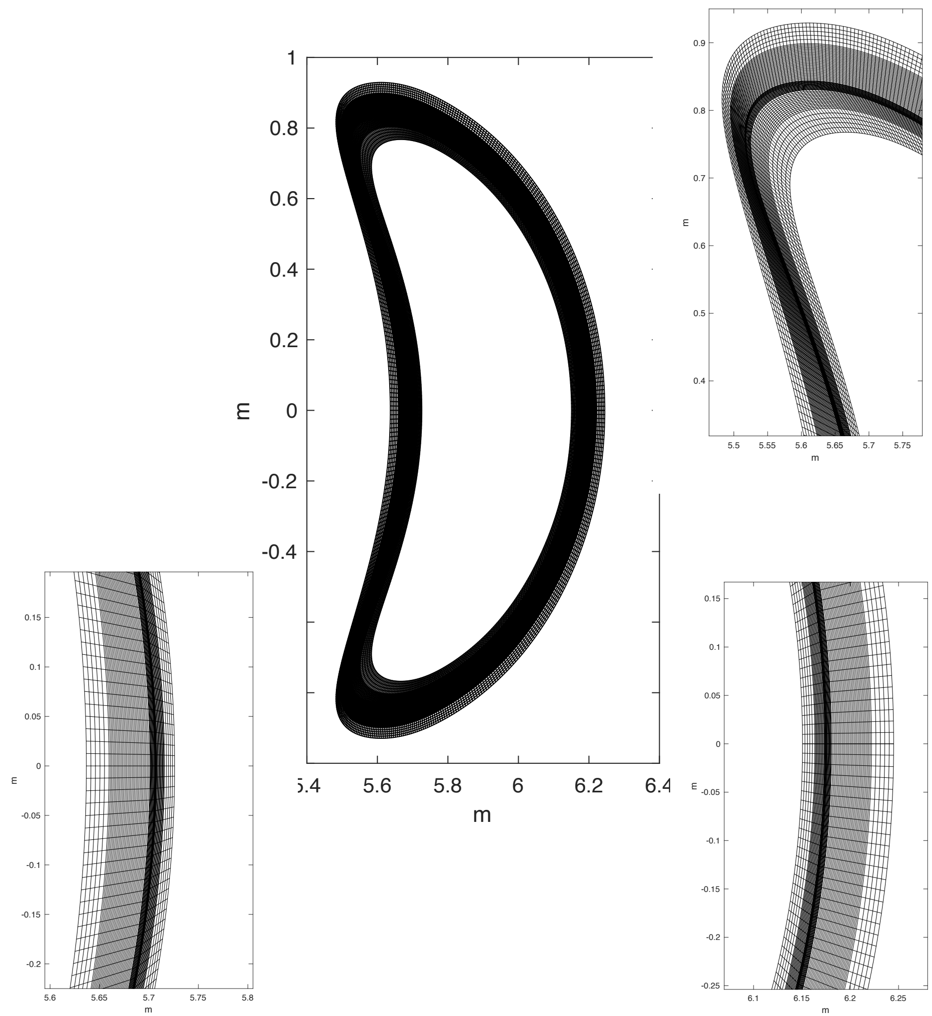
- The BMW code (M. Cianciosa) calculates the magnetic vector potential on a grid from a VMEC equilibrium using a volume integral over the plasma
- Magnetic field is then calculated from $\boldsymbol{B}(\boldsymbol{r}) = \nabla \times \boldsymbol{A}(\boldsymbol{r})$
- Grids are staggered to reduce numerical noise
  - Final grid has dimensions identical to the original MGRID used for the MHD equilibrium calculation by VMEC
- Divergence-free magnetic field is guaranteed

Codes used in this work

- BMW @ IPP (Magnetic Vector Potential, M. Cianciosa)
- EXTENDER_P @ IPP (Virtual Casing, M. Drevlak)
- FIELDLINES @ PPPL (Field-line following, S. Lazerson)
- FLF (Vacuum field & Field-line following, A. Bader)
- PARVMEC @ IPP and VMEC @ PPPL (MHD equilibrium calculation, Hirshman, et. al)
- get_VMEC_inverse_coords (Grid generation, J. Schmitt)

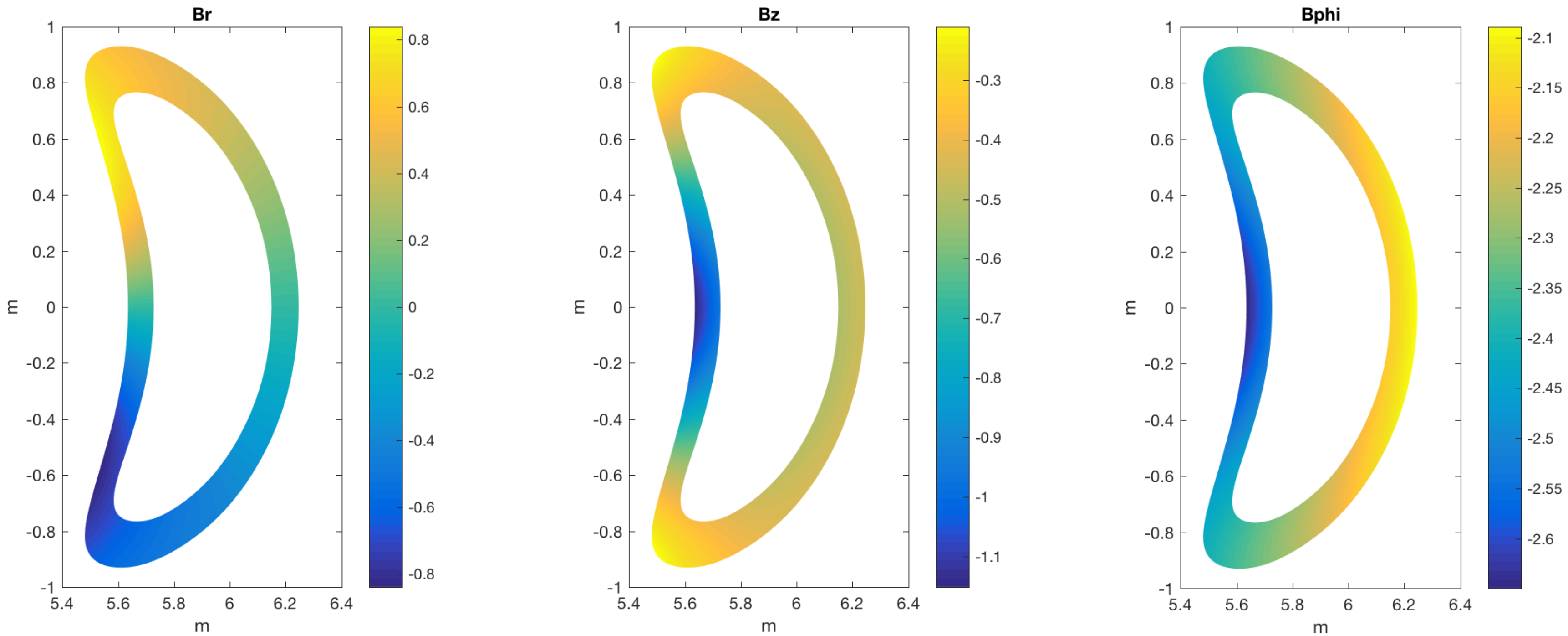
A dense mesh near the last closed flux surface was generated

- A dense mesh inside and outside of the vacuum last closed flux surface was generated
- The components of **B** are calculated on this dense mesh by the various methods and compared
- 36 toroidal ‘slices’ per field period are used, only the $\Phi = 0$ slice is shown here



Vacuum field on the grid

- Surface plots of the components of the magnetic field are shown for reference
  - Biot-Savart Law in vacuum (with FLF)



Vacuum field vs. 0% Beta Case
Virtual casing method

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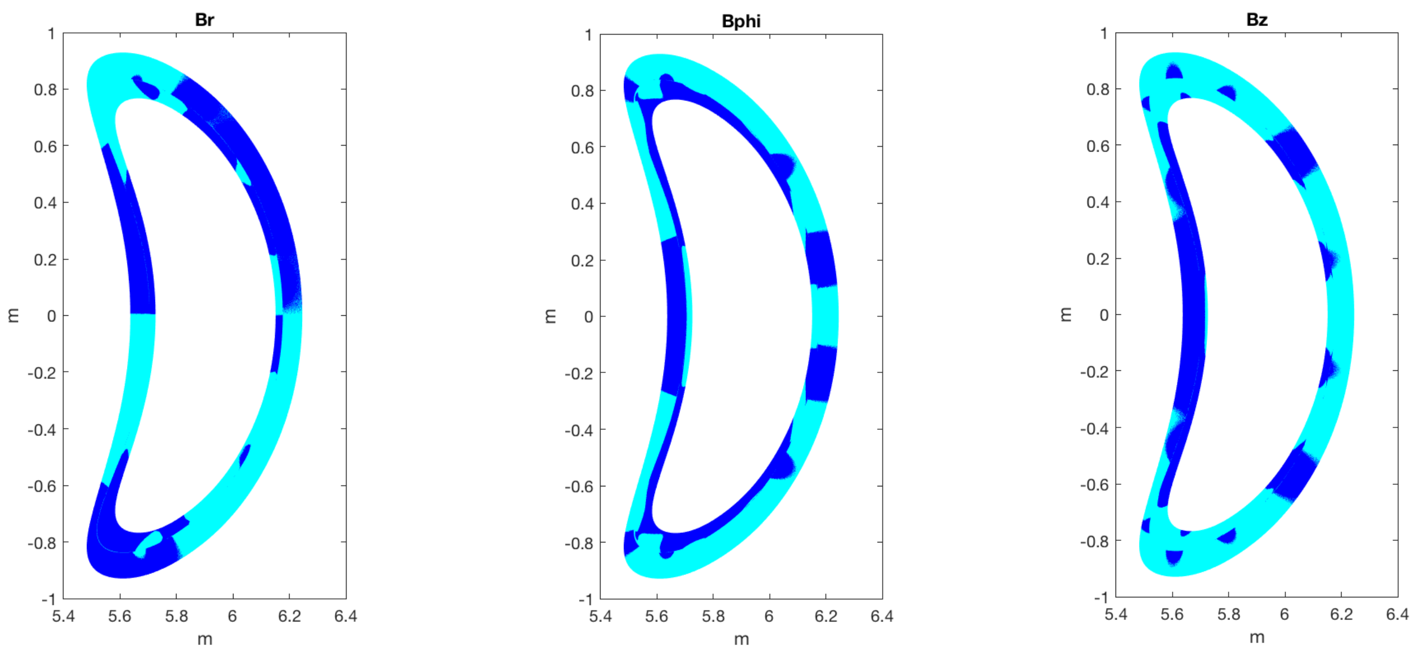
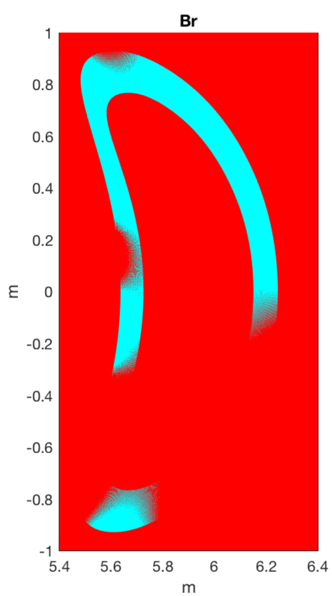
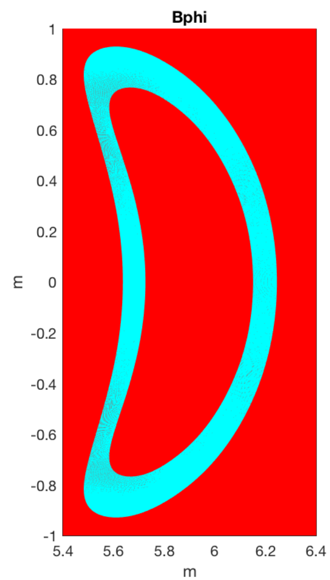
- The field components in vacuum and calculated for a 0% Beta VMEC calculation demonstrate the presence of finite current densities in the plasma region
- Here, the vacuum fields are plotted in cyan and the total field, form the Virtual Casing method are plotted in blue
- Surface current at the last-close flux surface demonstrates a structure that varies with VMEC convergence constraints (NS, NTOR, MPOL, FTOL)

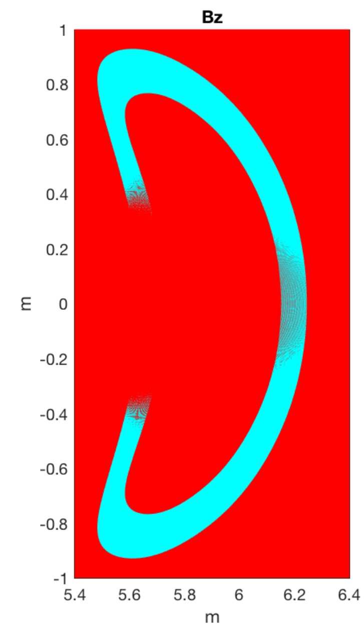


Vacuum field vs. 0% Beta Case Magnetic Vector Potential method

- The field components in vacuum and calculated for a 0% Beta VMEC calculation demonstrate a reduced sensitivity to the presence of finite current densities in the plasma region
- Here, the vacuum fields are plotted in cyan and the total field, form the Magnetic Vector Potential method are plotted in red
- Surface current at the last-close flux surface does not appear to have a large effect – Divergence-free magnetic field appears to be satisfied (better)

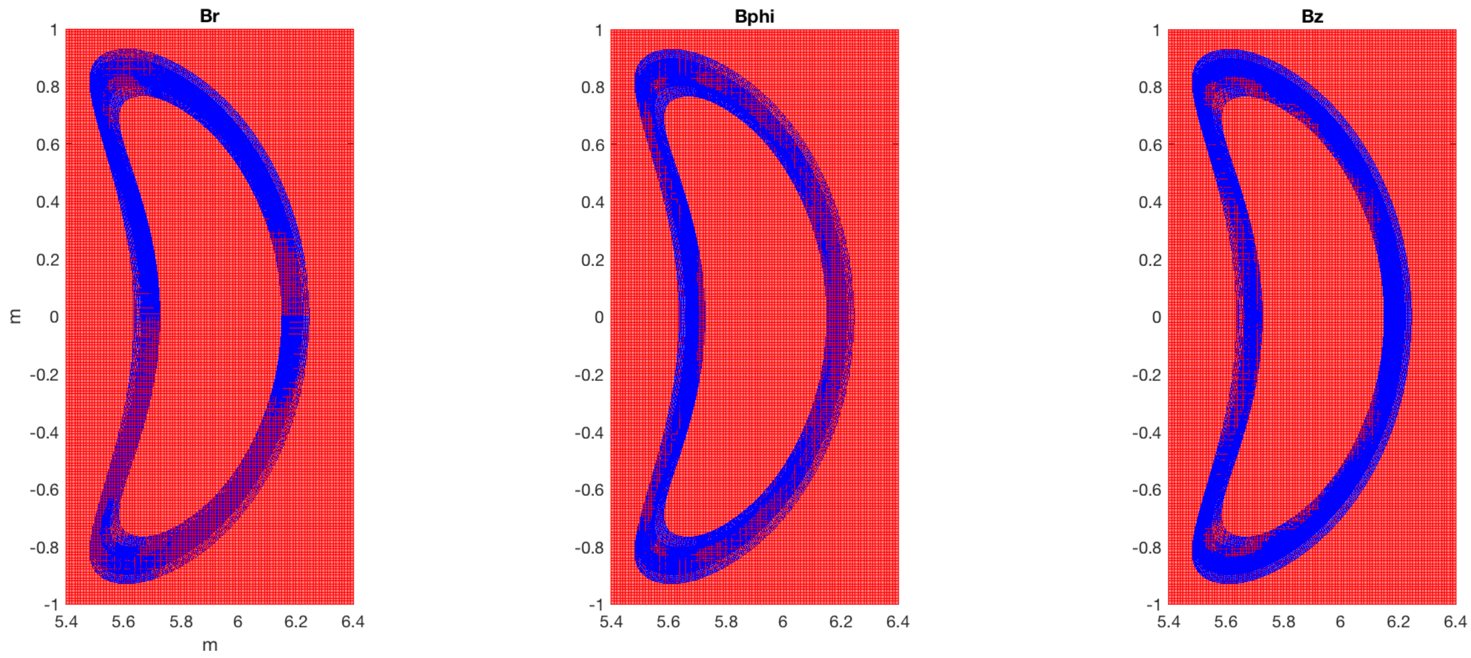






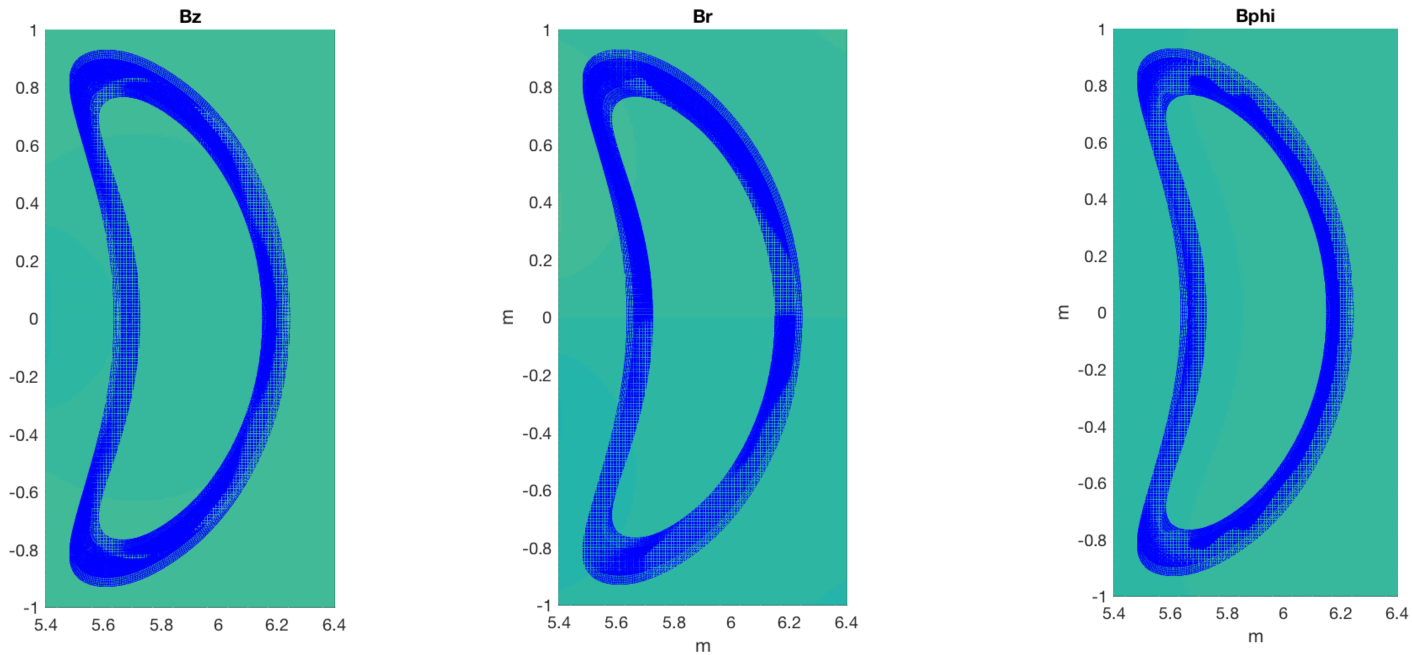
0% Beta Case
Virtual Casing vs Magnetic Vector Potential

- A direct comparison of the two methods can not be performed due to the difference in grid definitions, but overlaying the two surfaces in false color reveals similar information as shown in the VC vs Vacuum field comparison
- Finite plasma currents and the shielding current at the last closed flux surface both play a role



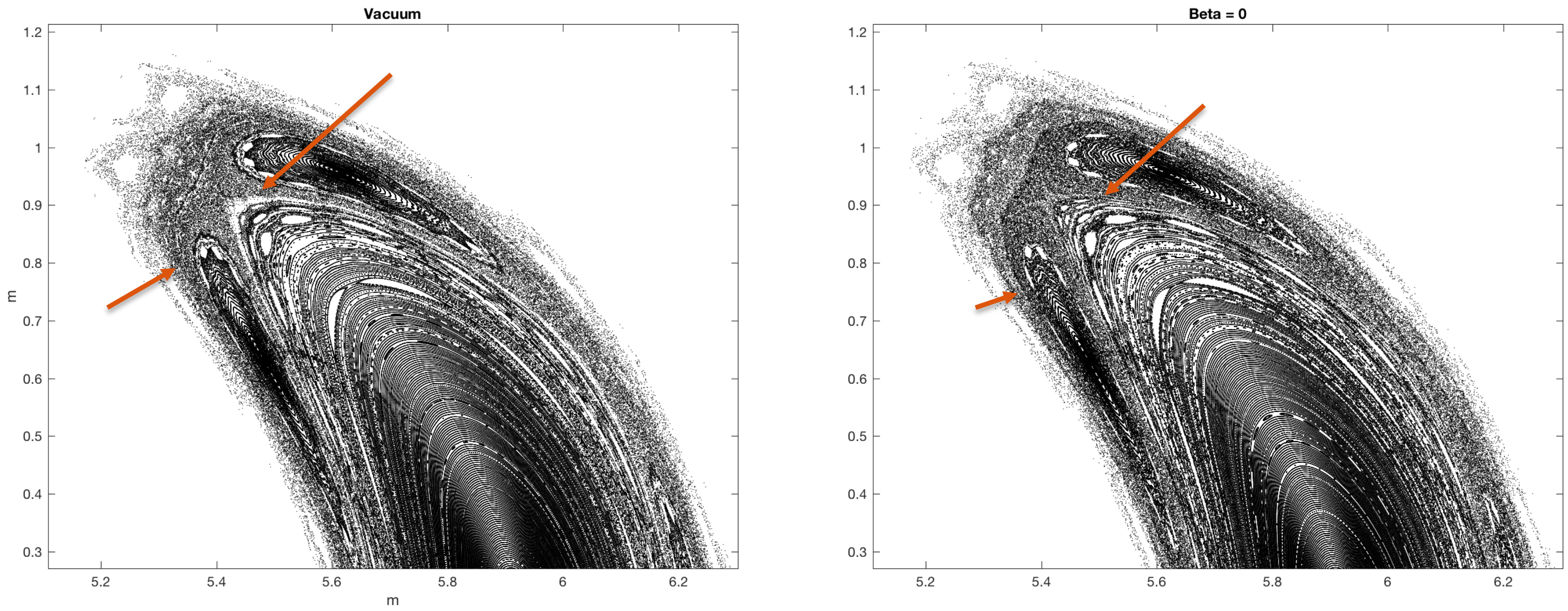
5% Beta, 5kA Case
VC vs Magnetic Vector Potential

- At finite beta and with net toroidal current differences between the two methods vaies, but still demostrates the effect of the shielding current at the last closed flux surface



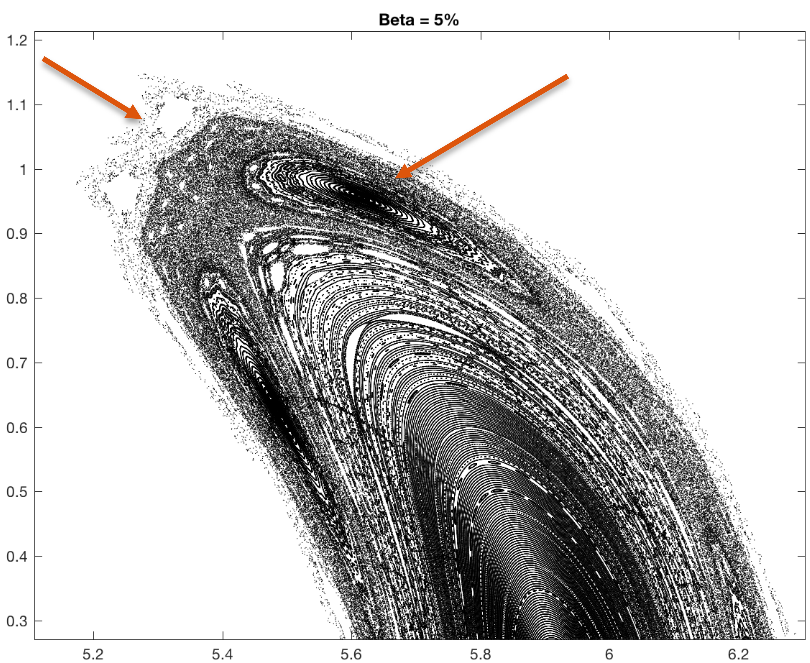
Field-line following with the total field as calculated with the vector potential

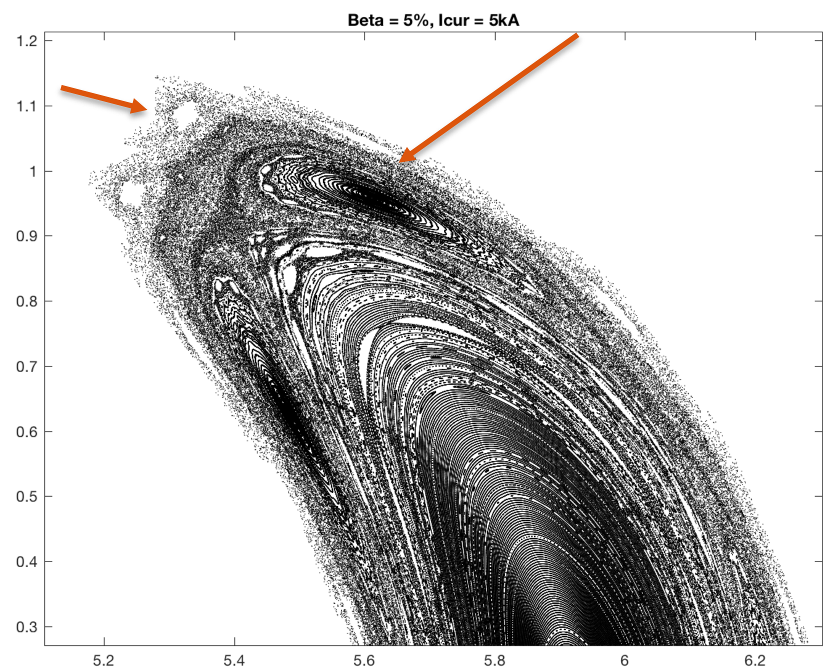
- Difference between the vacuum case and with 0% Beta case exist, but are small and subtle
  - Not limited close to the last closed flux surface
  - No large-scale numerical effects are present



As beta increases and toroidal current is introduce, edge region evolves

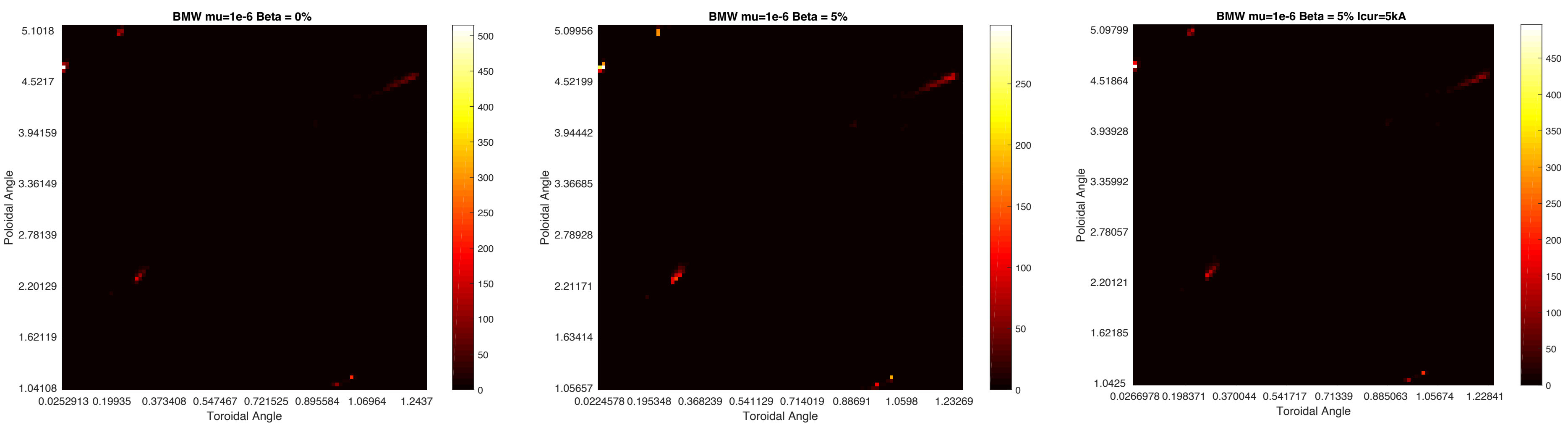
- Island structure and stochastic regions change with increasing toroidal current
- Expected to be significant at higher currents





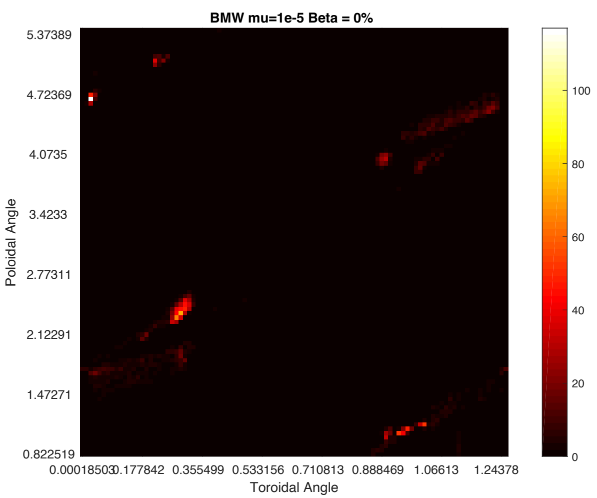
Strike points calculations vary with differing fields and diffusivities

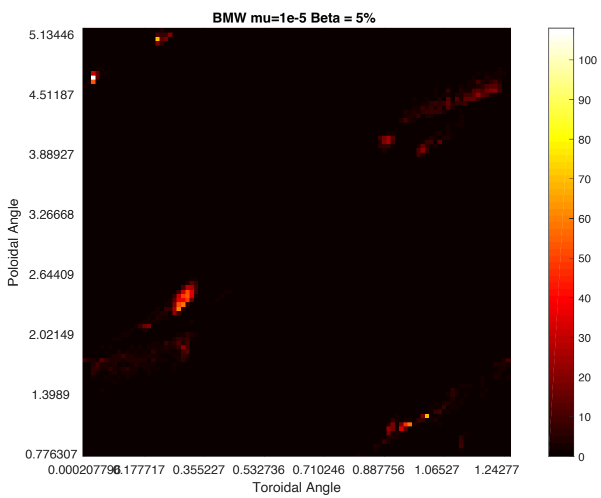
- Using the FIELDLINES code with finite diffusivity
- At low diffusivity (1e-6) strike line locations are quite similar, regardless of plasma conditions
- Number of strikeline incidents vary significantly

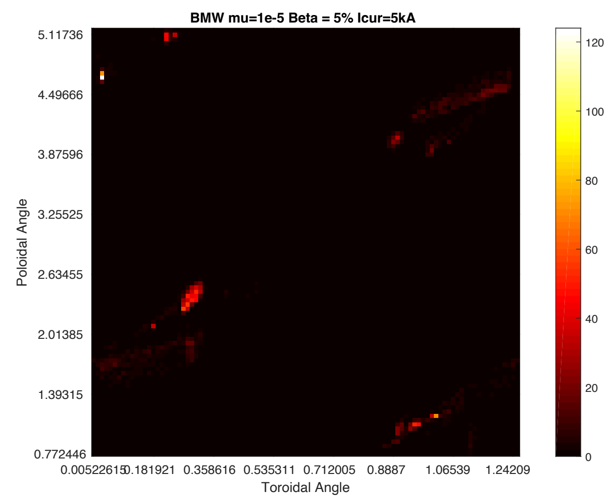


Increasing the field line diffusity

- Increasing the field line diffusivity (1e-5) begins to spread the strikeline locations across the divertor region
- Structure and location is similar; Small differences in the intensity

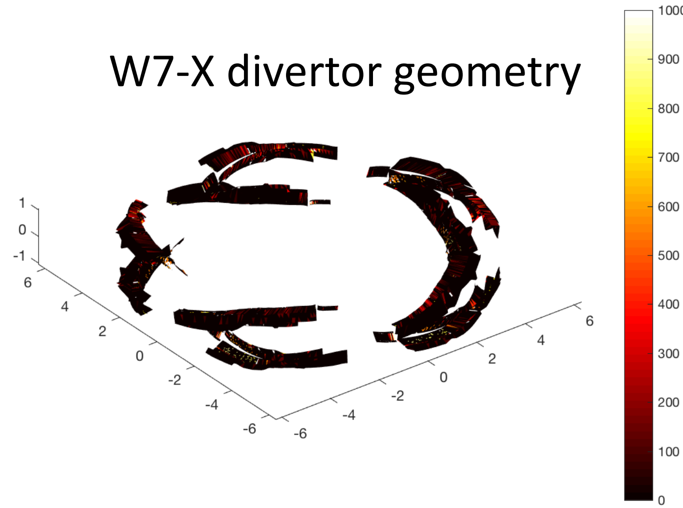


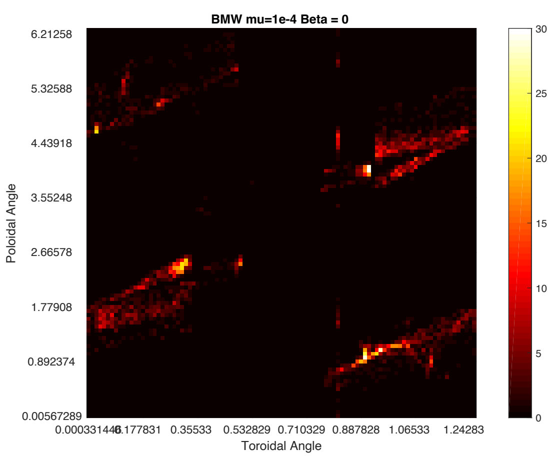


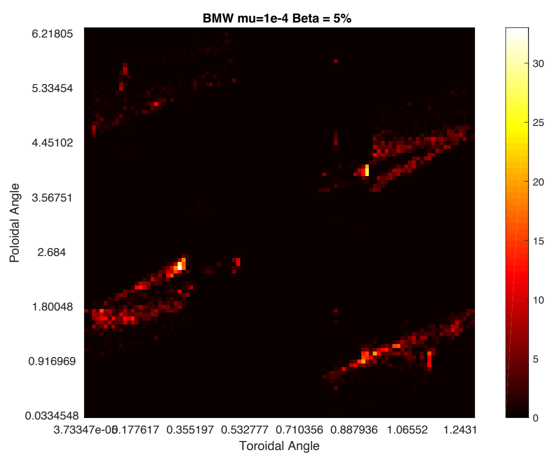


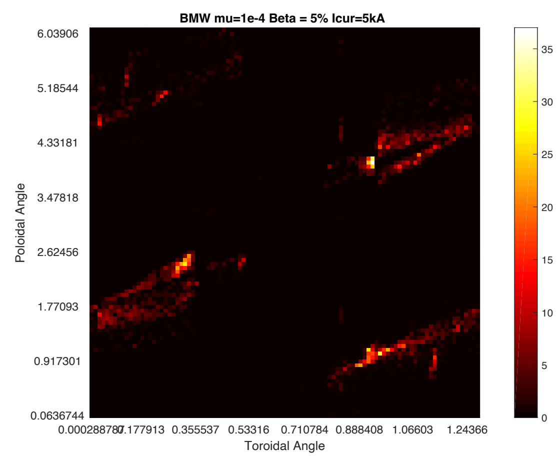
Increasing the diffusivity (more)

- As the diffusivity is increased further (1e-4), strikeline location continue to spread out, with only minor differences between various beta and current cases studied to date









Conclusion and Next steps

- Magnetic field calculations in the presence of plasma currents has been completed with multiple methods
  - Virtual casing theorem – Dense mesh
  - Magnetic vector potential – MGRID-style grid
- Field-line following with the BMW grid is successful and does not appear to introduce any numerical artifacts near the last-closed flux surface
- Learn to use the FLARE[1] code to build grid(s) for EMC3-EIRENE based on actual V3FIT reconstruction equilibrium

[1]H. Frerichs, et. al, http://meetings.aps.org/link/BAPS.2015.DPP.GP12.58 (2015).