Improving ideal ballooning stability limits and coil winding surfaces with STELLOPT

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Abstract

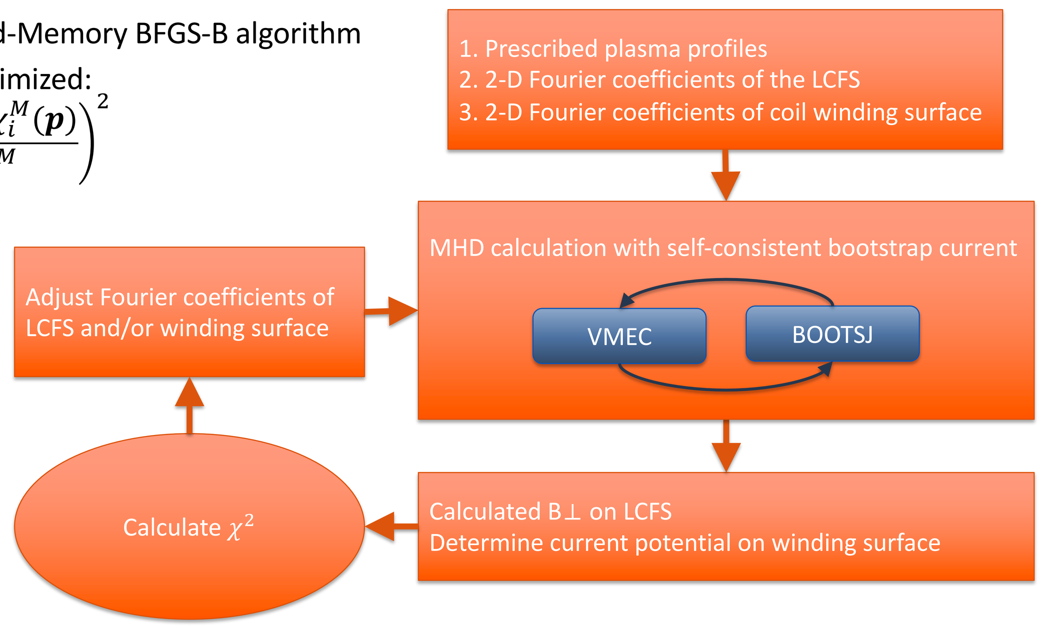
Optimized stellarator configurations will need to demonstrate desirable physics properties while simultaneously exhibiting a practical set of discrete coils, along with additional infrastructure and support structures necessary for the robust and stable operation of the device. The work here explores the use of the stellarator optimization suite, STELLOPT, to search for configurations that are predicted to have 1) better ideal MHD ballooning stability limits, and 2) improved coil winding surfaces that may lead to coils with improved engineering metrics. The ideal MHD ballooning stability calculation, based on a calculation requiring only the VMEC output data, is provided by COBRA. A rapid calculation of the coil winding surface current potential, the residual |Bnorm| error and a measure of the distances between coils, is provided by REGCOIL. By using metrics from both codes in the STELLOPT cost function, configurations with improved winding surfaces (reduced |Bnorm| error) and higher stability limits are found. Initial results of the optimization of quasi-symmetric stellarator configurations and candidate coil metrics will be shown.

Optimization Procedure

- Non-Linear Optimization procedure that adjusts the Fourier spectrum of the plasma boundary and coil winding surface to find configurations with improved characteristics
- Levenberg-Marquardt trust-region or Limited-Memory BFGS-B algorithm
- Cost function quantifies the target to be minimized:

$$\chi(\boldsymbol{p})^2 = \sum_i \left(\frac{\chi_i^T - \chi_i^M(\boldsymbol{p})}{\sigma_i^M} \right)^2$$

χ_i^T: Target function value
$\boldsymbol{p}$: Parameters that describe plasma equilibrium and winding surface
$\chi_i^M(\boldsymbol{p})$: Model function evaluation
σ_i^M : Target weight



Plasma Profiles

- 2-Species (Hydrogen) plasma

Temperature: $T_i(s) = T_e(s) = T_0\,(1-s)$
Density profiles $n_i(s) = n_e(s) = 7{\times}10^{19}\,(1-s^5)\, m^{-3}$
Radial Coordinate: $s \equiv \psi/_{\psi_{LCFS}}$, $\psi_{LCFS} = PHIEDGE$, $g^{ss} = |\vec{\nabla s}|^2$

- Bootstrap current

Analytic expression of bootstrap current in the collisionless limit[1]
Self-consistent profiles calculated via an internal loop in STELLOPT

Mercier Criterion

Mercier stability is given by[2,3]: $D_M = D_S + D_W + D_I + D_G \geq 0$

- Stabalizing shear term: $D_S \,\frac{\iota^2\pi^2}{s} = \frac{(\Psi''\Phi')^2}{4}$
- Well (or hill) term: $D_W \,\frac{\iota^2\pi^2}{s} = \int\int g d\theta d\zeta \frac{B^2}{g^{ss}}\frac{dp}{ds} \times \left(V'' - \frac{dp}{ds}\int\int g \frac{d\theta d\zeta}{B^2}\right)$
- Net current: $D_I \,\frac{\iota^2\pi^2}{s} = \left[\int\int g \frac{d\theta d\zeta}{g^{ss}}\frac{B^2}{g^{ss}}\Psi'' I' - (\Psi''\Phi')\int\int g \frac{(\boldsymbol{J}\cdot\boldsymbol{B}) d\theta d\zeta}{g^{ss}}\right]$
- Geodesic curvature: $D_G \,\frac{\iota^2\pi^2}{s} = \left[\int\int g \frac{(\boldsymbol{J}\cdot\boldsymbol{B}) d\theta d\zeta}{g^{ss}}\right]^2 - \left[\int\int g \frac{(\boldsymbol{J}\cdot\boldsymbol{B})^2 d\theta d\zeta}{g^{ss}B^2}\right]\int\int g \frac{B^2 d\theta d\zeta}{g^{ss}}$

Ballooning Stability

- Ideal ballooning mode growth rates are rapidly and accurately calculated using VMEC coordinates[4]

$$\alpha(s,\theta_v,\zeta_v) = \theta_v + \lambda(s,\theta_v,\zeta_v) - \iota\zeta_v$$
$$(\vec{B}\cdot\nabla)\left[\frac{|\nabla\alpha|^2}{B^2}(\vec{B}\cdot\nabla)\right]F + \left(\frac{R_0}{a}\right)^2 \frac{\beta_0 p'}{\Psi'^2}\kappa_s F = \gamma^2 \frac{|\nabla\alpha|^2}{B^2} F$$

Coil Winding Surface

- Normal component of B *on the plasma surface:* $\chi_B^2 = \int d^2a\, B_\perp^2$
- Current density *on the winding surface* is given by $\vec{\boldsymbol{K}} = \hat{n}\times\nabla\Phi$
- Surface−average−squared current density $K^2 = |\boldsymbol{K}|^2$ *on the winding surface:* $\chi_K^2 = \int d^2a' K^2$
- REGCOIL[5] finds the current potential, Φ, on the winding surface which minimizes: $\chi^2 = \chi_B^2 + \lambda\chi_K^2$
- Regularization parameter, λ: Large regularization leads to less complicated current potential contours ('less harmonic content'), but χ_B^2 increases

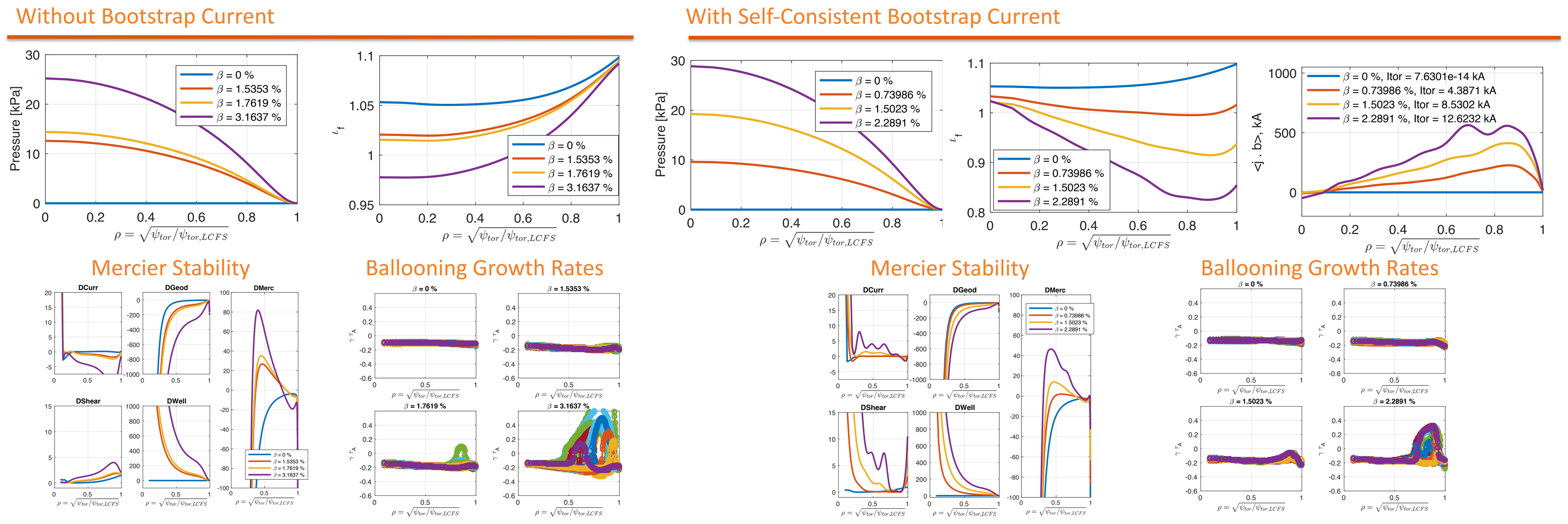
- REGCOIL cost function targets available for STELLOPT (**bold** indicates targets used in this work)

$\boldsymbol{\kappa_{MAX}}$, $\boldsymbol{\kappa_{RMS}}$, χ_K^2
$\boldsymbol{\chi_B^2}$, $\boldsymbol{B_\perp(\theta,\zeta)}$, $\boldsymbol{max(B_\perp)}$
Minimum distance between plasma and winding surface
Volume (plasma, **winding surface**, or difference)
Surface area (plasma, winding surface, or difference)

Configuration Comparisons

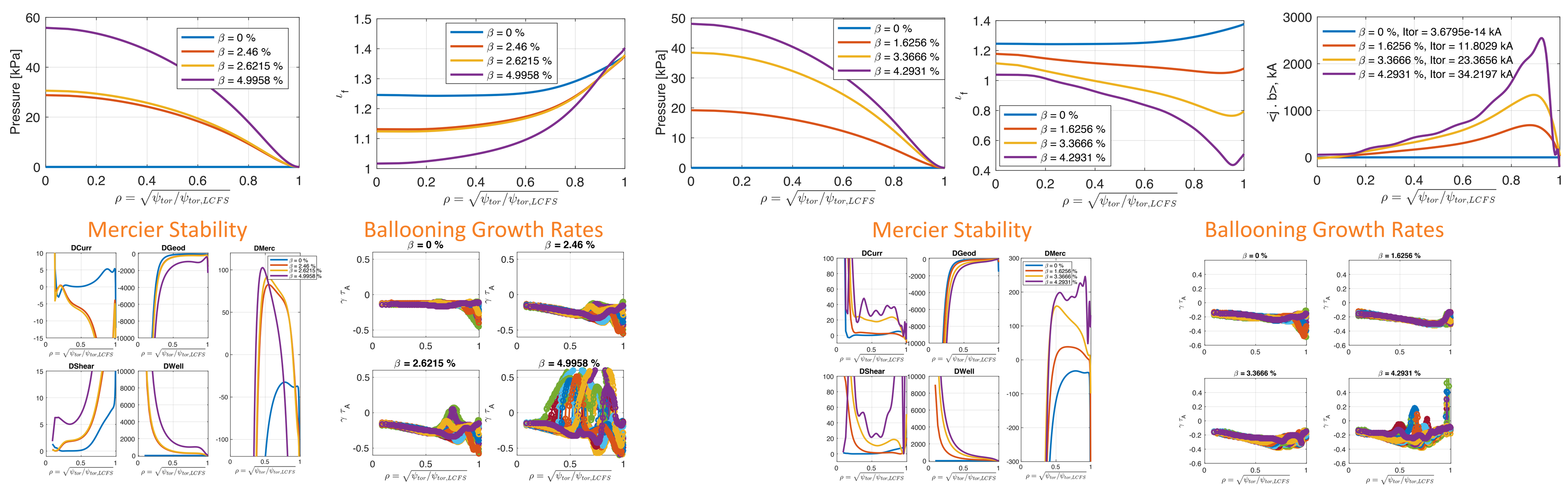
QHS

- Standard configuration of the Helically Symmetric Experiment (HSX)
- Straight-line B-field spectrum dominated by (n,m) = (4,1) helical component
- Vacuum transform from 1.05 – 1.10
- Bootstrap current *unwinds/lowers* the transform
- Mercier stability improves with beta due to deeper well
- Ideal ballooning stability limit between β=1.5% - 1.7% (with or without bootstrap current)



Well (10%)

- A 'high iota' configuration of the Helically Symmetric Experiment (HSX) with increased well depth (stability)
- Vacuum transform from 1.25 – 1.35
- Ideal ballooning stability limit @ β = 2.5% (without bootstrap current) or >3.3% (with bootstrap current)

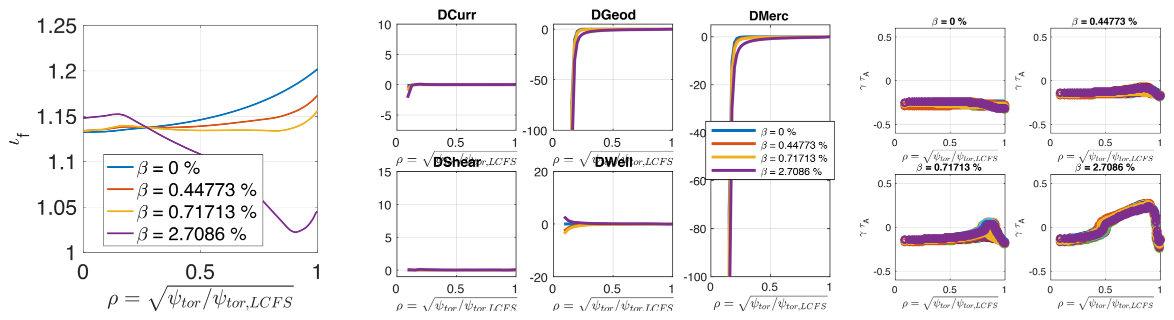


WISTELL CANDIDATES

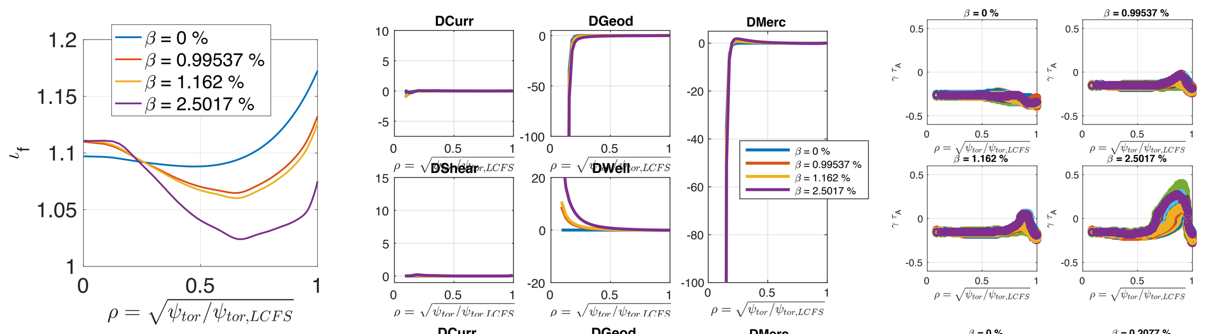
- Several preliminary configurations are under consideration for in-depth analysis and (possible) optimization.
- Self-consistent bootstrap current included in this analysis
- All configuration are ballooning unstable above β = 1.1%
- Mercier criterion convergence needs to be confirmed.
- Configurations have either 4 or 5 field periods

$\frac{8}{7} \approx 1.14$	$\frac{10}{7} \approx 1.43$
$\frac{12}{11} \approx 1.09$	$\frac{15}{11} \approx 1.36$
$\frac{16}{15} \approx 1.07$	$\frac{5}{4} = 1.25$
$\frac{16}{17} \approx 0.94$	$\frac{15}{13} \approx 1.15$
$\frac{12}{13} \approx 0.92$	$\frac{10}{9} \approx 1.11$
$\frac{8}{9} \approx 0.88$	

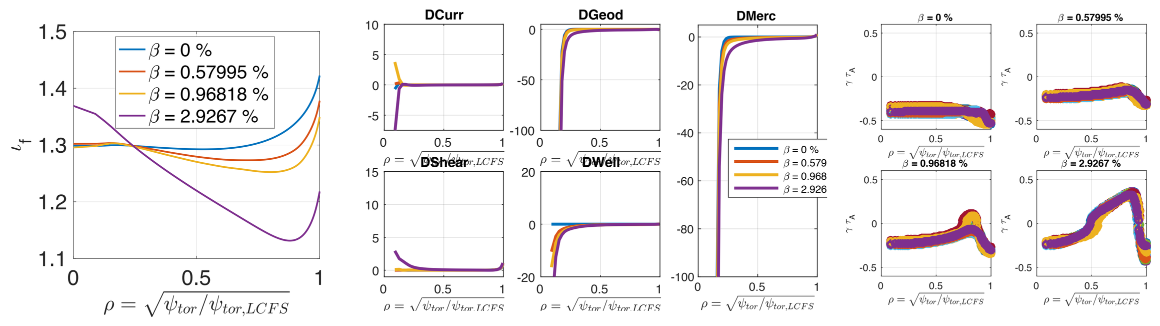
MLJS3



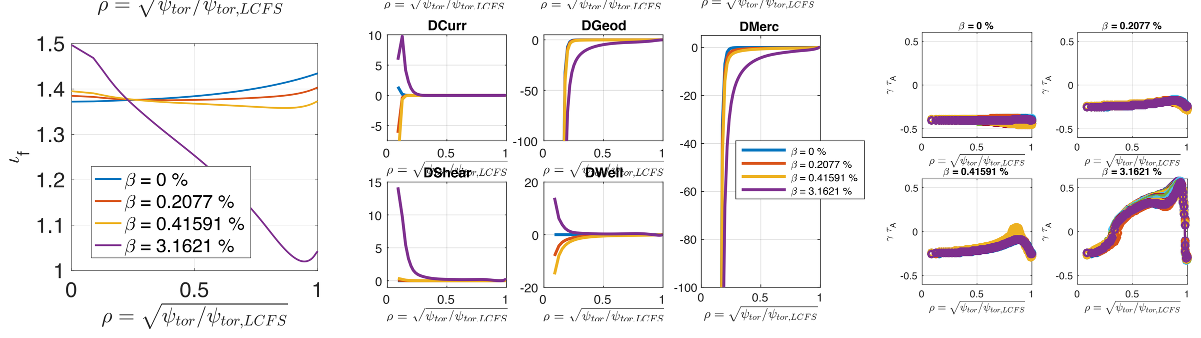
ATEN



BILA (5 FP)



MLJS4 (5 FP)



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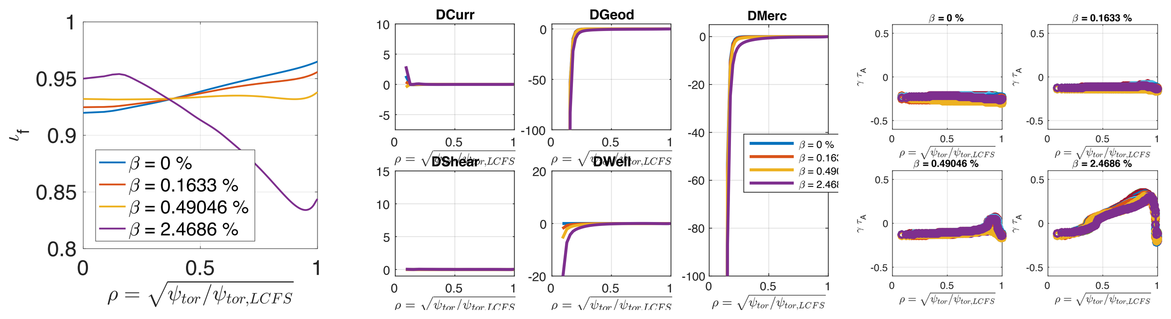
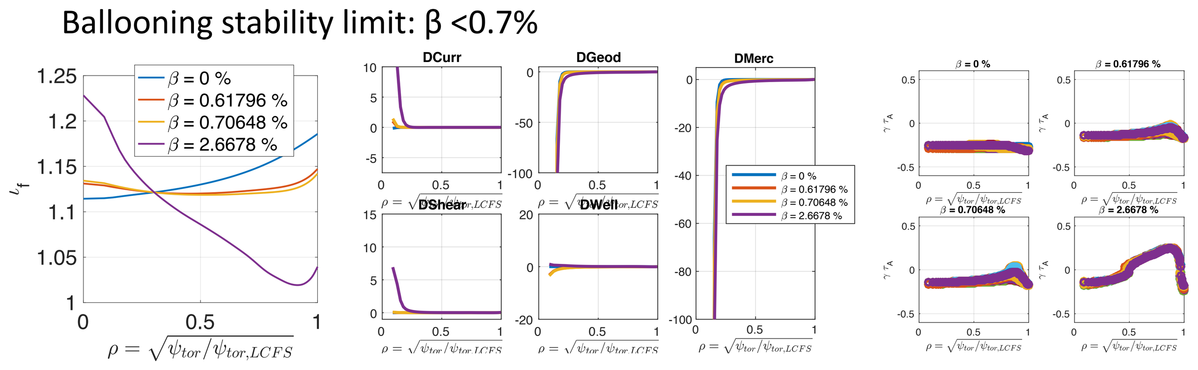


Prior to Optimization

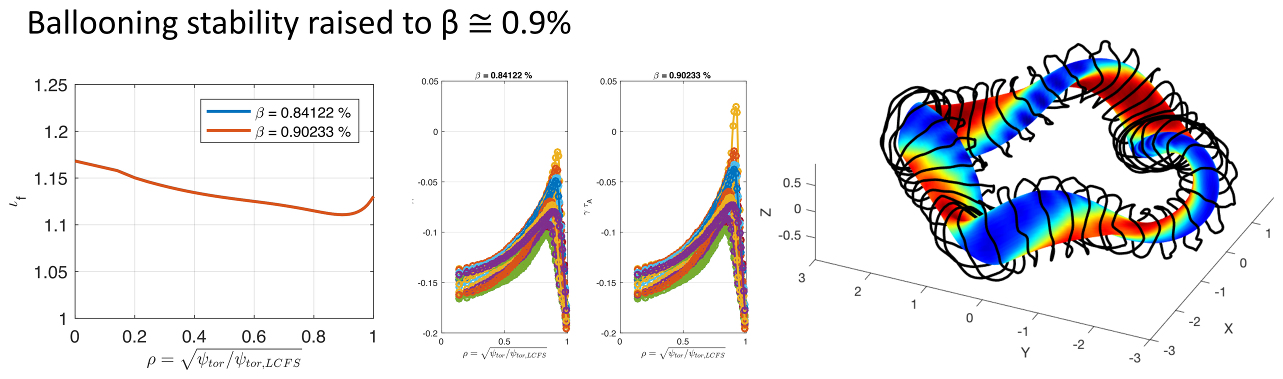
Ballooning stability limit: β <0.7%

MLJS2



After Optimization

Ballooning stability raised to β ≅ 0.9%

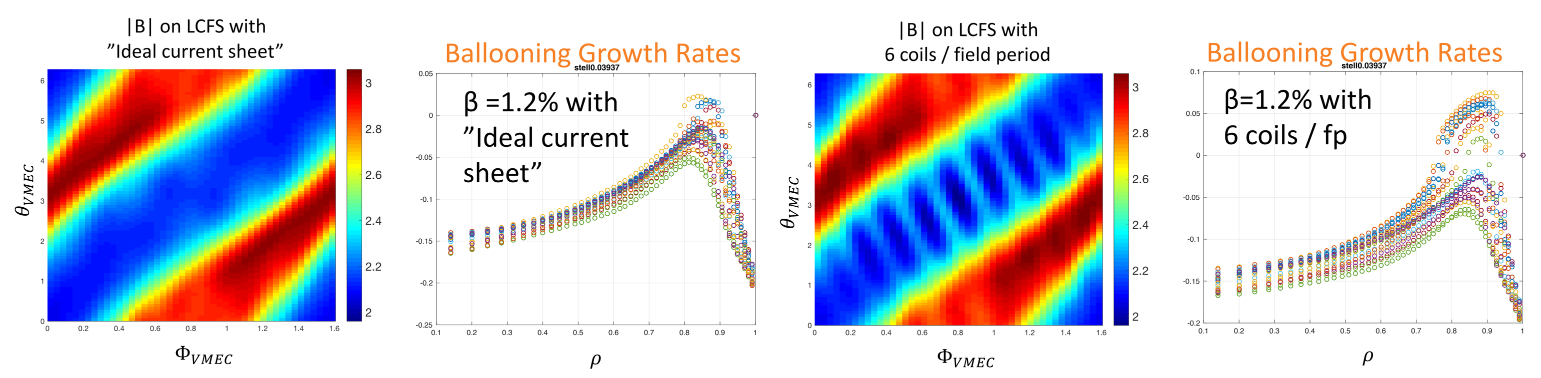


Coil Metrics (Single filament model)

Coil-to-coil distance (min): 16.5 cm
Coil length (avg): 3.76 m
Toroidal extent (avg): 0.348 rad (20 °)
Curvature (mean arclength): 2.97 m^{-1}

Coil length (max): 3.89 m
Toroidal extent (max): 0.387 [rad] (22 °)
Curvature (max arclength): 12.1 m^{-1}

- Discrete coils produce a noticeable coil ripple on inboard low-field side
- An increase in the predicted growth rates is also observed



Conclusions and Future Work

- Simultaneous optimization for ballooning stability and coils metrics on a winding surface is feasible in STELLOPT
- Other metrics ($\boldsymbol{\Gamma}_C$, turbulence, etc.) can be included in the standard way with STELLOPT
- Can be adapted to work for permanent magnet solutions
- Coils can be further improved (reduced coil ripple, improved metrics) with FOCUS
- All six configurations are under analysis

References

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